Fundamental Frequency Estimation by Augmented Complex Least Mean Squares in Single-Phase Power Grids

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***Abstract*—In this paper, it is proposed an adaptation of the augmented complex least mean squares (ACLMS) method, originally conceived for three-phase systems, to estimate the fundamental frequency in single-phase grids. It relies on the second-order generalized integrator (SOGI) structure that is able to provide two output voltages in quadrature out of one sinusoid voltage input. The quadrature voltages are then used to build a rotating complex vector which is the ACLMS input. Moreover, some improvements are suggested to make the proposal robust against harmonics and faster for estimating the frequency. Due to its simple realization, the proposed algorithm is suitable for real-time applications. In this paper, the method and its improvements are evaluated with simulated signals emulating severe distortion conditions, and an experimental test is realized in laboratory. The results are promising and show the usability and effectiveness of the proposed method.**

***Index Terms*—Augmented Complex Least Mean Square, Gradient Descent, Frequency Estimation, Single-Phase Power Systems.**

I. Introduction

Nowadays, the increasing usage of distributed generators (DGs) greatly affects the power system in terms of operation of its operations and protection. One critical issue, within this context, is reversion of power flow through the grid. This is critical because the protection is traditionally conceived to work in a radial system with power flowing in one direction, from the substation to the loads [1]. The power flow reversion tends to turn the faults inflicting the grid stringently. In such situations, the fundamental frequency might experience undesirable changes that ought be monitored.

The knowledge of the grid frequency in real-time is paramount in functions such as power relaying, power-quality monitoring and control of power converters connected to the grid [2], [3]. Techniques based on zero-crossing and on the Fourier algorithms are commonly employed in different applications [4]. However their performance seriously degrade in the presence of voltage distortions. Alternatives like Kalman filter, neural networks and genetic algorithms have been proposed to deal with such distortions [5], [6]. A very interesting technique is the one based on the least mean squares due to its simplicity and suitability to real-time applications in three-phase systems. The generalization of such technique to complex signals and to single-phase systems is the focus of this paper.

The remaining sections are organized as follows: Section II describes the algorithms based on Least Mean Square; the proposed Augmented Complex Least Mean Square algorithm adapted to Single-Phase Grids are presented in Section III; Section IV is dedicated to the analysis of simulation results; and the final comments are in Section V.

II. Algorithms Based on Least Mean Square

The Least Mean Squares (LMS) is based on the algorithm proposed by Widrow and Hoff in 1960 [7]. Since its early proposal, it has been widely applied due to computational simplicity and robustness. Several improvements have been introduced in the last years, in this section, it is discussed two of them.

A. The Complex Least Mean Square Algorithm

The Complex Least Mean Square (CLMS) method is based on the classical least mean squares algorithm. The basic difference is that the input, unlike the LMS, is a complex signal. In [8] the algorithm is employed in three-phase systems and the complex voltage vector $\boldsymbol{v}$ is determined by

$$\mathbf{v} = v_\alpha + jv_\beta, \tag{1}$$

where v_α and v_β are, in turn, determined by the Clark's transformation applied on the three-phase grid voltages [9]. The calculation of the adaptive vector for frequency estimation is summarized in Fig. 1. This figure shows a model $\hat{\mathbf{v}}_{\mathbf{k+1}}$

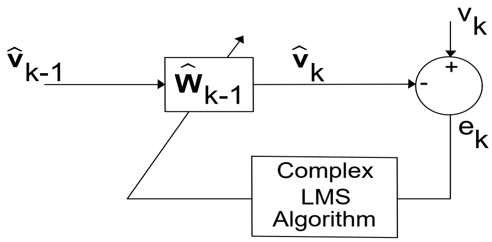

Figure 1: CLMS basic diagram.

which depends on the voltage given in the previous time step k that is filtered by an adaptive filter $\hat{\mathbf{W}}_{\mathbf{k}}$. Hence:

$$\hat{\mathbf{v}}_{k+1} = \mathbf{W}_k \hat{\mathbf{v}}_k. \tag{2}$$

Accordingly to the CLMS algorithm, one must consider the model (2) and the error $\mathbf{e}_k$, defined by:

$$\mathbf{e}_k = \mathbf{v}_k - \hat{\mathbf{v}}_k. \tag{3}$$

This error clearly depends on $\mathbf{W}_k$. The CLMS algorithm updates $\mathbf{W}_k$ by minimizing the quadratic error $\mathbf{\Psi}_k$:

$$\mathbf{\Psi}_k = \mathbf{e}_k \mathbf{e}_k^*, \tag{4}$$

where $*$ stands for the complex conjugate, through a gradient descent equation, given by:

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \mu_k \nabla_W \mathbf{\Psi}_k, \tag{5}$$

where $\nabla_W \mathbf{\Psi}_k$ is the gradient of $\mathbf{\Psi}_k$ and μ_k is a convergence factor which can be constant or updated at each sampling time, accordingly to the method describe in [10]. Developing (5), one reaches, after some manipulation:

$$\mathbf{W}_k = \mathbf{W}_{k-1} + \mu_k \mathbf{e}_k \hat{\mathbf{V}}_k^*. \tag{6}$$

This equation is a practical form to evolve the filter $\mathbf{W}$ with time. The estimated frequency must be updated accordingly to the evolution of $\mathbf{W}$. To compute an expression for the frequency, one has to take into account that in a balanced three-phase systems, the synchronous complex voltage $\mathbf{v}_k$ is described by a complex vector with constant value, rotating in the counter clockwise direction, that is:

$$\mathbf{v}_k = A e^{j(\omega k \Delta T + \phi)}, \tag{7}$$

where ω is the rotating frequency frequency equivalent to the grid frequency in radians, ΔT is the sampling period for the voltages and ϕ is an arbitrary phase. Considering (2) and (7), it is possible to devise that:

$$\mathbf{W}_k = e^{j\hat{\omega}\Delta T}. \tag{8}$$

Then, $\mathbf{W}_k$ can be rewritten as:

$$\mathbf{W}_k = \cos(\omega \Delta T) + j \sin(\omega \Delta T). \tag{9}$$

Manipulating (9), the estimated frequency $\hat{f}$ is given by:

$$\hat{f}[k] = \frac{1}{2\pi \Delta T} \arcsin(Img(\hat{\mathbf{W}}_k)) \tag{10}$$

B. The Augmented Complex Least Mean Square Algorithm

As already mentioned, the CLMS described above is conceived for estimating the fundamental frequency of balanced voltages. This fact is mathematically expressed by (7). For unbalanced grid conditions this model can no longer be applied. It is shown in [11] that the voltage vector $\mathbf{v}_k$ in such conditions is better described by:

$$\mathbf{v}_k = A(k) e^{j(\hat{\omega} k \Delta T + \phi)} + B(k) e^{-j(\hat{\omega} k \Delta T + \phi)} \tag{11}$$

where $A(k)$ e $B(k)$ are given by:

$$A(k) = \sqrt{6} \frac{v_a(k) + v_b(k) + v_c(k)}{6} \tag{12}$$

and

$$B(k) = \sqrt{6} \frac{v_a(k) - v_b(k) - v_c(k)}{12} - j\sqrt{2} \frac{v_b(k) - v_c(k)}{4}. \tag{13}$$

Observing (11), it is suggested in [11] that a model for $\mathbf{v}_k$ can be written as:

$$\hat{\mathbf{v}}_{k+1} = \mathbf{v}_k \mathbf{h}_k + \mathbf{v}_k^* \mathbf{g}_k, \tag{14}$$

where $\mathbf{h_k}$ and $\mathbf{g_k}$ are adaptable weighting vectors that are determined by the LMS algorithm, similarly to the procedure explained in the last section. The updating expressions are:

$$\mathbf{h}_k = \mathbf{h}_{k-1} + \mu \mathbf{e}_{k-1} \mathbf{v}_{k-1}^*, \tag{15}$$

and

$$\mathbf{g}_k = \mathbf{g}_{k-1} + \mu \mathbf{e}_{k-1} \mathbf{v}_{k-1}. \tag{16}$$

Note that (14) replaces (2) and (11) replaces (7) for unbalanced conditions.

The grid frequency can then be computed by manipulating (14) and (11), with a procedure analogous to the one discussed in the last section. The frequency is given by:

$$\hat{f}[k] = \frac{1}{2\pi \Delta T} \arcsin(Im(\mathbf{h}_k + \mathbf{a}_{1k} \mathbf{g}_k)), \tag{17}$$

where

$$\mathbf{a}_{1k} = \frac{-jIm(\mathbf{h}_k) + j\sqrt{Im^2(h_{\mathbf{k}}) - |\mathbf{g}_k|^2)}}{\mathbf{g}_k}. \tag{18}$$

C. Variable-Step-Size

The convergence factor μ is key variable for LMS algorithm convergence [12], [13]. LMS algorithms that employs constant μ have deteriorated performance in the presence of noise [14]. In this sense, LMS algorithm with a variable step size (VSS-LMS) for μ has been devised in [14]. Accordingly, the convergence factor is updated at each instant with:

$$\mu_{k+1} = \alpha \mu_k + \gamma p_k^2 \tag{19}$$

where, $0 < \alpha < 1$, $\gamma > 1$ and p_k is an estimative for the autocorrelation between e_k and e_{k-1} given by:

$$p_k = \beta p_{k-1} + (1-\beta)(e_k e_{k-1} + e_k^2), \tag{20}$$

with β being a weighting positive constant limited as β $(0 < \beta < 1)$. The variable convergence factor has its values limited to assure convergence, accordingly to:

$$\begin{cases} \mu_{max}, & \text{if} \mu_k > \mu_{max} \\ \mu_{min}, & \text{if} \mu_k < \mu_{min} \\ \mu_k, & \mu_{min} < \mu_k < \mu_{max} \end{cases} \tag{21}$$

III. The Augmented Complex Least Mean Square Algorithm Adapted to Single-Phase Grids

This paper proposes to adapt the ACLMS algorithm to estimate the fundamental frequency in single-phase power grids. To this end, one proposes to add to it, the SOGI structure depicted at Fig. 2. This structure is able to dynamically provides two artificial quadrature voltages, v_α and v_β from a sinusoid voltage, v_i. As it can be noted, the SOGI structure consists of two integrators. The relevant transfer functions related to the SOGI are $G_d(s)$ and $G_q(s)$. Then, $G_d(s)$ is given as:

$$G_d(s) = \frac{v_\alpha}{v_i}(s) = \frac{k_p \omega^{'} s}{s^2 + k_p \omega' s + \omega'^2}, \tag{22}$$

where $\omega^{'}$ is the SOGI resonant frequency and k_p is a gain, usually set in $\sqrt{(2)}$ to better dynamic execution [15]. Posteriorly, $G_q(s)$ is obtained through:

$$G_q(s) = \frac{v_\beta}{v_i}(s) = \frac{k_p \omega^{'2}}{s^2 + k_p \omega' s + \omega'^2}, \tag{23}$$

Both transfer functions $G_d(s)$ and $G_q(s)$ are resonant frequencies for $0 \leq k_p < 2$ able to sort out the component of v_i in frequency $\omega^{'}$, with no gain. The difference is that while $G_d(s)$ produces no delay, $G_q(s)$ lags v_β of 90 degrees. In power grid applications, $\omega^{'}$ is set in the frequency fundamental value [15].

The overall basic schematic proposed in this paper is depicted in Fig. 3. It is depicted in four stages. The first one, the SOGI acts on the input voltage v_i to produce the quadrature signals v_α and v_β. In this version of the method, the SOGI receives a fixed frequency $\hat{\omega}_0$ assumed to be the real grid fundamental frequency $\hat{\omega}_0$. In the second stage, the

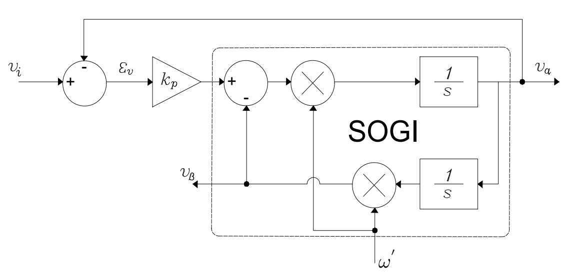

Figure 2: Second-order generalized integrator (SOGI) structure.

complex vector is formed through (1). In the third stage, the complex signal is processed by the ACLMS algorithm through (14), (16) and (15). However, an adaptable convergence factor, given by (19), is used. Hence, (16) and (15) are rewritten as:

$$\mathbf{h}_k = \mathbf{h}_{k-1} + \mu_k \mathbf{e}_{k-1} \mathbf{v}^*_{k-1}, \tag{24}$$

and

$$\mathbf{g}_k = \mathbf{g}_{k-1} + \mu_k \mathbf{e}_{k-1} \mathbf{v}_{k-1}. \tag{25}$$

The last stage is the frequency computation. The ACLMS algorithm computes the weighting vectors $\mathbf{h_k}$ and $\mathbf{g_k}$ that are used to calculate the fundamental frequency through (17).

Alternatively, instead of having a fixed frequency furnished to to the SOGI, the frequency computed by the ACLMS ca be supplied back to the SOGI, as it is shown in Fig. 4. This is designated A-ACLMS-FBF.

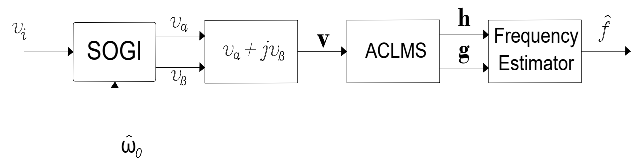

Figure 3: Adapted augmented complex least mean algorithm with fixed frequency (A-ACLMS-FF) for frequency estimation in single-phase grids.

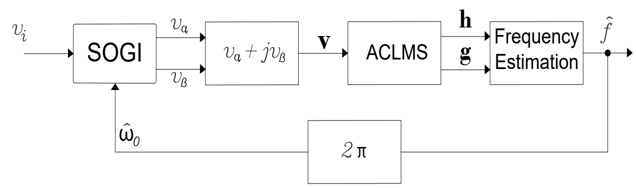

Figure 4: ACLMS feedback frequency (A-ACLMS-FBF) for frequency estimation in single-phase grids.

Finally, the last alternative with regards the SOGI frequency is based on the well-known frequency locked loop (FLL) [16]. Unlike the other approaches, in this one, the frequency is computed by the FLL block that is incorporated to the SOGI structure, as shown in Fig. 6. This alternative is designated as A-ACLMS frequency locked loop (A-ACLMS-FLL) and it is shown in Fig. 5.

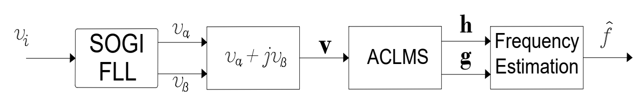

Figure 5: ACLMS frequency locked loop (A-ACLMS-FLL) for frequency estimation in single-phase grids.

IV. Results

In this section, three proposed approaches, A-ACLMS-FF, A-ACLMS-FBF, and A-ACLMS-FLL are compared with traditional FLL, and the CLMS technique, also adapted to

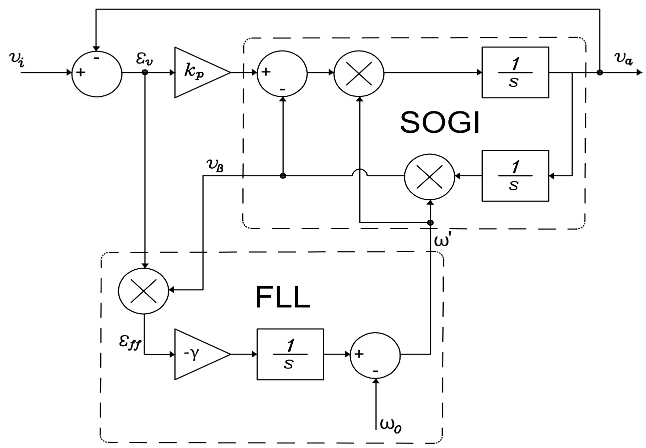

Figure 6: SOGI-FLL structure.

single-phase grids. It is considered two situations for testing the techniques. The first serves to evaluate the algorithms when they are applied to a voltage in which the frequency suddenly changes from 60 to 62 Hz. In the second one, the same test is repeated, however the input signal is contaminated with harmonics.

To accomplish the tests, it is necessary to single out the parameters accordingly to [8], [11], [15], [17], [18]. These parameters are shown in Tab. I.

To compare the techniques, a set of figures of merit to quantify their performance, in respect to [19]. These figures are the settling time (ST), the overshoot (OS) and the stationary error (SE) for 60 and 62 Hz. With regards the settling time, one has been established a range formed by the frequency value $\pm 0.25\%$ in which the estimation must accommodate.

Fig. 7 shows the input voltage with amplitude of 1 p.u. and frequency of 60 Hz that changes to 62 Hz in 0.53 s. Fig.8 show the frequency estimation for five methods: The CLMS estimation, the A-ACLMS-FF estimation, the A-ACLMS-FBF estimation, the A-ACLMS-FLL estimation and the FLL estimation. This last estimation is the one computed by the FLL block that is incorporated to the SOGI in the A-ACLMS-FLL approach. At first, as shown in Fig. 8, the FLL frequency estimation presents greater oscillation with regards the others. The second part (when the frequency changes), it is noticeable an oscillation increasing of the CLMS estimation. The other techniques show no significant oscillation, notably the A-ACLMS-FF technique that performs the best. Tab.II summarize the results. It is evident a pronounced difference in the SE for the CLMS estimation.

TABLE I: Parameters

Parameters	
Variables	***Values***
μ_{min}	0.007
μ_{max}	0.012
λ	0.08
α	0.97
β	0.99
k_p (SOGI's gain)	$\sqrt{2}$
γ (FLL's gain)	46

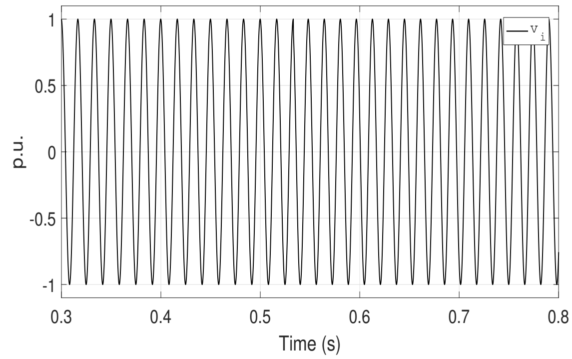

Figure 7: Input signal without harmonics.

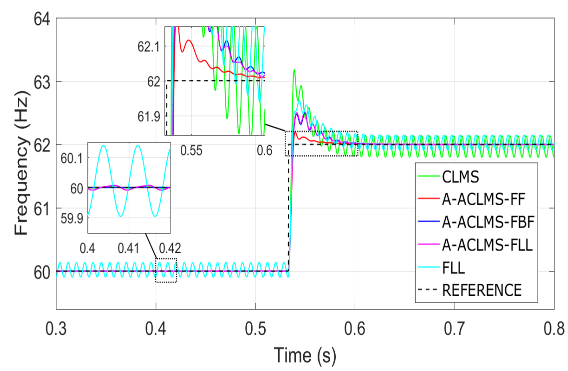

Figure 8: Frequency step in 0.53 seconds without harmonics.

The second test is carried out with the same voltage as the first test but with the addition of a fifth, seventh and the eleventh harmonics, with amplitudes of 0.06. The voltage is depicted in Fig.9. Fig 8 shows the frequency tracking for the same six methods of the first test. It is visible the oscillation for all of them. Nevertheless, the lesser is oscillation is presented by the ACLMS estimation.

Likewise, the previous test, the CLMS method increases the oscillation when the frequency changes. Tab III summarizes the results for the second test. It is notable that all methods have their frequency estimation degraded with regards the previous case. However only the CLMS and FLL estimations are off the bounds imposed to the (frequency value $\pm 0.25\%$).

TABLE II: Results Without Harmonics

Methods	***ST (ms)***	***OS (%)***	***SE-60Hz (Hz)***	***SE-62Hz (Hz)***
CLMS	inf	1.91	$2\text{x}10^{-3}$	0.2
A-ACLMS-FF	8.0	0.32	10^{-10}	$2\text{x}10^{-7}$
A-ACLMS-FBF	34.4	0.84	$8\text{x}10^{-10}$	$5\text{x}10^{-8}$
A-ACLMS-FLL	34.0	0.85	$7\text{x}10^{-3}$	$8\text{x}10^{-3}$
FLL	84.5	1.12	0.14	0.15

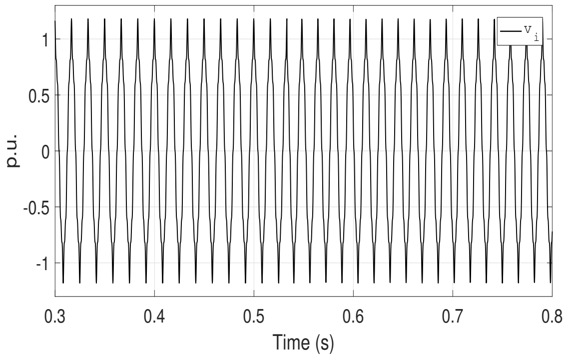

Figure 9: Input signal with harmonics.

TABLE III: Results With Harmonics

Methods	*ST (ms)*	*OS (%)*	*SE-60Hz (Hz)*	*SE-62Hz (Hz)*
CLMS	inf	1.9	0.17	0.32
A-ACLMS-FF	37.0	0.34	0.136	0.11
A-ACLMS-FBF	66.0	1.02	0.128	0.135
A-ACLMS-FLL	57.0	1.05	0.123	0.13
FLL	inf	1.20	0.17	0.17

Finally, for complete evaluation of the methods involved, it conducted a test with an extracted signal experiment in real time. Fig. 11 shows the experimental setup of a DG system able to generate voltage signal. The system is composed by Texas Instruments testing module with DSP controller (TMDSHV1PHINVKIT). The system also has an auxiliary source providing DC voltage for powering the testing module, the voltage bender circuit that works in conjunction with the variable transformer to deliver a stable voltage to the inverter terminals and finally isolating transformer,which serves to connect the inverter output to the grid.

For this frequency step test is used experimental voltage signal shown in Fig. 12. The frequency change occurs in 0.23 seconds as shown in Fig. 13, where is realized the estimated frequency for a real signal. This figure shows small variations of about 0.2 Hz for A-ACLMS-methods (FF, FBF and FLL), and up to 0.6 and 0.8 Hz for the CMLS and FLL methods,

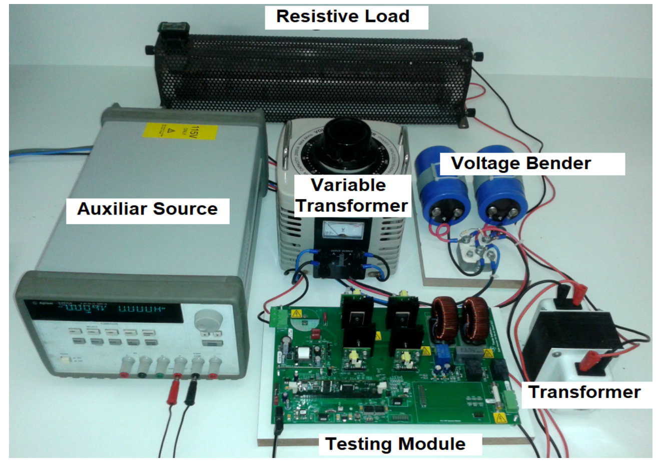

Figure 11: Experimental setup.

respectively.

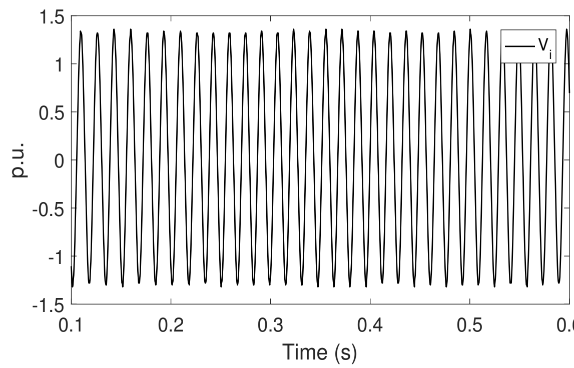

Figure 12: Real input signal.

V. Conclusion

This paper proposes an adaptation of the augmented complex least mean squares, originally developed to frequency estimation in three-phase power systems, for estimation frequency in single-phase grids. The adaptation is based on

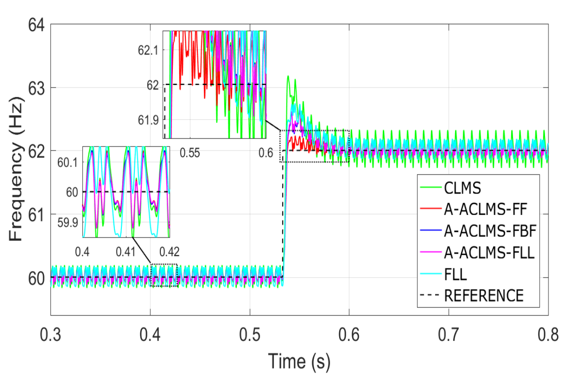

Figure 10: Frequency step in 0.53 seconds with harmonics.

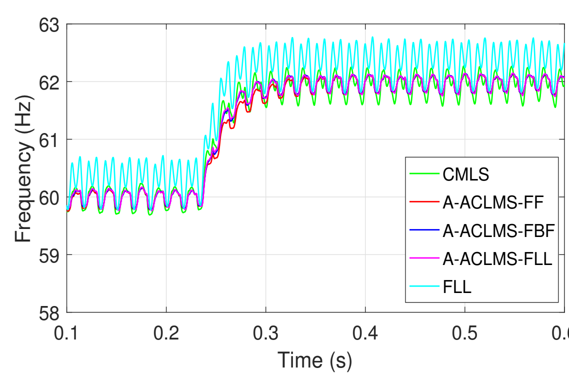

Figure 13: Frequency step in 0.23 seconds for a real signal.

the so-called second-order generalized integrator technique to produce quadrature voltages. With these voltages, it is possible to build a vector whose rotating frequency, the same as the grid frequency, is estimated by the ACLMS. This method varies with the form in which the frequency is provided to the SOGI. Three schemes are briefly presented in this work. The options are to furnish the SOGI with a fixed frequency, a feedback frequency or a frequency computed by a structure called frequency locked loop. The results without harmonics show that the proposed techniques can effectively track the a frequency that is suddenly modified. In comparison with the CLMS and FLL algorithms, the techniques are faster to converge and present lower oscillations in steady-state. In the presence of harmonics all the methods display significant oscillations. The ACLMS methods, however, manages to keep the estimation within the limits for steady-state. Finally, an experimental test was realized to demonstrates that the ACLMS methods, presenting a better effectiveness in relation to CLMS and FLL.

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